DYNAMICS OF VAPOR BUBBLES IN NITROGEN TETROXIDE UNDER CONDITIONS OF PIPE DEPRESSURIZATION

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A study was made of the effect of nonequilibrium phase transformations on the dynamics of vapor bubbles with the sudden occurrence of a pressure drop.

Vapor–liquid mixtures are most commonly used as heat-transfer agents in heat and power engineering and nuclear power plants. This accounts for the considerable interest in the study of the structure of two-phase vapor–liquid flows with different flow regimes [1, 2], the generation of shock waves [3, 4] and rarefaction waves in channels [5], and the effect of liquid–vapor phase transformations accompanying these processes on the structure and properties of the entire system.

Since the properties of two-phase gas–liquid systems differ appreciably from the properties of pure liquids and vapors and depend to a large extent on the dynamics of the vapor or gas bubbles present, then determination of the gas dynamic parameters in each phase of a gas–liquid mixture requires study of the dynamics of the bubbles, i.e., of their behavior with a change in the external parameters of the system. It is also necessary to study the interaction of bubbles with the surrounding liquid. Several studies [2-10] have offered a systematic survey of the results of theoretical investigations of the dynamics of spherical and nonspherical gas bubbles in a liquid in relation to the type of effects (inertial, thermal, or diffusive) that are predominant. The studies [2-7] were devoted to numerical investigation of the interaction of a gas bubble with a liquid. Allowing for phase transformations with oscillations of vapor bubbles complicates the solution of the problem even if it is assumed that the parameters inside the bubble are uniform [6, 7, 11-13]. The studies [12, 13] presented results of a numerical analysis of the dynamics of a spherical vapor bubble in water with the assumption that pressure inside the bubble was uniform. The authors used a finite-difference approximation of the problem to reduce it to a system of ordinary differential equations. The Runge–Kutta method was used to obtain a solution.

Here, we study cavitation processes occurring in a liquid when there is a pressure drop in it. Such processes take place in pipes filled with a liquid heat-carrier when the pipe is depressurized. We will make the usual assumptions employed in studying the dynamics of a single bubble: spherical symmetry is maintained, the density of the vapor at each point corresponds to its temperature at the given pressure in accordance with the equation of state $P = A\rho^{\gamma}$. The system consists of the equations of energy, continuity, and motion for the vapor and liquid phases [6, 13]:

$$C_{pv}\rho_v\left(\frac{\partial T_v}{\partial t}+V_v\frac{\partial T_v}{\partial r}\right)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda_v r^2\frac{\partial T_v}{\partial r}\right)+\frac{dP_v}{dt},\quad 0\leqslant r\leqslant R, \tag{1}$$

$$V_v=\frac{(\gamma-1)\lambda_v}{\gamma P_v}\frac{\partial T_v}{\partial r}-\frac{r}{3\gamma P_v}\frac{dP_v}{dt},\quad 0\leqslant r\leqslant R, \tag{2}$$

$$\frac{dP_v}{dt}=3(\gamma-1)\lambda_v\frac{1}{r^2}\frac{\partial T_v}{\partial r}\bigg|_{r=R}-3\gamma\frac{P_v}{r}W_v, \tag{3}$$

$$C_l\rho_l\left(\frac{\partial T_l}{\partial t}+V_l\frac{\partial T_l}{\partial r}\right)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda_l r^2\frac{\partial T_l}{\partial r}\right),\quad r>R, \tag{4}$$

$$V_l=W_l\frac{R^2}{r^2}, \tag{5}$$

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$$R\frac{dW_l}{dt}+\frac{3}{2}W_l^2+2j\frac{W_l}{\rho_l}=\frac{P_v-P_l+\frac{2\sigma}{R}}{\rho_l}-4\nu\frac{W_l}{R}. \tag{6}$$

The velocities of the bubble boundary dR/dt and the mass velocities of the phase at this boundary are connected by the relations:

$$\frac{dR}{dt}=W_l+\frac{j}{\rho_l}, \tag{7}$$

$$\frac{dR}{dt}=W_v+\frac{j}{\rho_v}. \tag{8}$$

The equations expressing the change in the mass and density of the bubble have the form:

$$\frac{dm}{dt}=4\pi R^2 j; \tag{9}$$

$$\rho_v=\frac{3}{4\pi}\frac{m}{R^3}. \tag{10}$$

System (1-10) is solved with the following initial and boundary conditions:

$$t=0:\ P_v=P_{v0},\ \rho_v=\rho_{v0},\ R=R_0,\ W_l=W_{l0},\ T_v=T_l=T_0,\ P_l=P_{l0}, \tag{11}$$

$$\frac{\partial T_v}{\partial r}=V_v=0,\quad r=0, \tag{12}$$

$$\lambda_l\frac{\partial T_l}{\partial r}-\lambda_v\frac{\partial T_v}{\partial r}=jl,\quad r=R, \tag{13}$$

$$T_l|_{r=+\infty}=T_\infty, \tag{14}$$

$$T_v=T_l=T_s(P_v),\quad r=R. \tag{15}$$

The problem was solved in spherical coordinates in the varibles $\xi = r/R(t)$, t. For convenience in calculating the internal and external problem, we introduce the new variable

$$x=\begin{cases}\xi, & \xi\leqslant 1,\\ 2-\dfrac{1}{\xi}, & \xi\geqslant 1.\end{cases}$$

The method of calculation is as follows: At the initial moment of time, it is assumed that the liquid–vapor-bubble system is in equilibrium, and equilibrium values of all functions of system (11) are assigned for the given temperature and pressure. The pressure gradient ($P_V - P_\ell$) is assigned and we use an explicit scheme with a half time step to calculate values of temperature $T^{(1/2)}$ at all specified coordinate points. The values of $T_i^{(1/2)}$ are used to calculate P_V, ρ_V, R, and W_ℓ and their time derivatives on the "half" time layer. These values are then used to calculate the main parameters on the next integral time layer. Meanwhile, the temperature T on the integral layer is calculated by an implicit scheme. To check the integration on each time layer, we assign an error EPS < 1% for recalculation of the main parameters P_V, R, ρ_V, and W_ℓ by the "rough" scheme (i.e., with calculation of the parameters by an explicit scheme with an integral time step) and the working schemes on one time layer. The step of the time grid should be changed if EPS > 1%. To calculate the saturation temperature and the heat of phase transformation at the given pressure, we used the interpolation formulas [14]:

$$T_s(P)=1\Big/\left(a_T\ln\frac{P}{P_0}+B_T\right),$$

$$l(P)=l_0\exp\left[\left(a_p+b_p\ln\frac{P}{P_0}\right)\ln\frac{P}{P_0}\right].$$

The coefficients a_T, B_T, a_p, b_p are calculated for specified tabular values.

As the test substance for the calculations, we used nitrogen tetroxide. The values of its parameters were taken from [15].

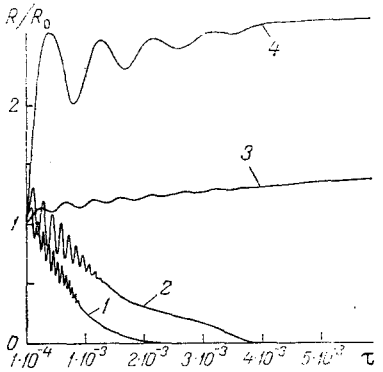

Fig. 1

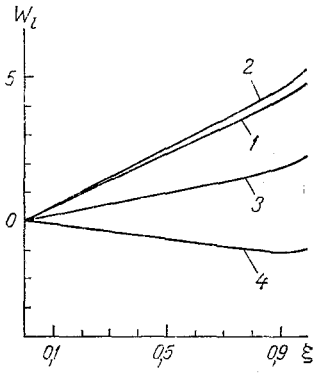

Fig. 2

Fig. 1. Dependence of the dimensionless radius of a vapor bubble on time at the temperature T = 300 K for a pressure drop in the liquid from P_0 to P_ℓ: 1) $P_0 = 1.5 \cdot 10^6$ Pa; $P_\ell = 1 \cdot 10^6$ Pa; 2) $1 \cdot 10^6$ and $0.5 \cdot 10^6$; 3) $1 \cdot 10^5$ and $0.8 \cdot 10^5$; 4) $1 \cdot 10^6$ and $1 \cdot 10^5$. τ, sec.

Fig. 2. Velocity profile inside the bubble at $T_0 = 300$ K and pressure drops from 10^6 to $5 \cdot 10^5$ Pa at different moments of time: 1) $\tau = 0.25 \cdot 10^{-4}$ sec; 2) $0.5 \cdot 10^{-4}$; 3) $0.75 \cdot 10^{-4}$; 4) $0.1 \cdot 10^{-3}$ sec. W_ℓ, m/sec.

We performed calculations of the cavitational process occurring with an instantaneous pressure drop in a liquid far from a bubble from P_0 to P_ℓ, which corresponds to the case of the depressurization of a pipe filled with a heat carrier at P_0 in a vacuum with the pressure P_ℓ. Figure 1 shows the behavior of a vapor bubble with an initial radius $R_0 = 0.001$ m in liquid nitrogen tetroxide N_2O_4 heated to below the boiling point ($T_0 = 300$ K) for different pressure drops. It is evident that for cavitation to occur, not only is the pressure drop from P_V to P_ℓ important, but so is the initial value P_V. A lower initial value P_V leads to growth of the vapor bubble with a pressure drop even of $0.2 \cdot 10^5$ Pa when $P_V = 1 \cdot 10^5$ Pa (curve 3), while an initial value $P_V = 1 \cdot 10^6$ Pa requires a pressure drop of almost one order (curves 1 and 4) for cavitation to occur. High values of P_V with an insufficiently large pressure drop lead to collapse of the vapor bubbles (curves 1 and 2). The energy released with collapse of the bubbles may either be dissipated due to the viscosity and thermal conductivity of the liquid or may be converted into the kinetic energy of the liquid. Calculations performed for different temperatures and pressure drops show that an increase in the initial temperature of the system significantly intensifies the development of cavitation (with an identical pressure drop). Pulsations of the vapor bubble are one consequence of the vaporization and condensation occurring inside it. The pulsations stop after an equilibrium value $P_V = P_\ell + 2\sigma/R$ is established. Here, the bubble then either grows monotonically or, if the equilibrium value of the radius becomes less than a certain critical value R_{cr}, it collapses. Given the same pressure drop, an increase in the initial temperature of the process lengthens the time interval during which the bubble pulsates. The intensity of these oscillations depends on the initial size of the bubble. Small bubbles ($R_0 < 0.001$ m) are characterized by monotonic growth, while both the intensity and duration of the pulsations increase with an increase in the initial size of the bubble.

Calculations were performed in the temperature range 300-333 K. Experimental data for nitrogen tetroxide is available in this range. The measurement methodology and some results were presented in [5, 16, 17]. The occurrence of cavitation processes in a pipe when it undergoes depressurization in a vacuum can be detected both from oscillograms showing the change in the system pressure and from the change in and the intensity of monochromatic light passing through the vapor–liquid medium. The growth of cavitational bubbles leads to an increase in the volume of the vapor phase in the vapor–liquid medium, and this in turn increases the intensity of the light passing through the medium. Calculated dependences of the change in vapor pressure P_V and bubble radius R on time agree qualitatively with experimental oscillograms recorded with the same initial temperatures and identical values of pressure drop.

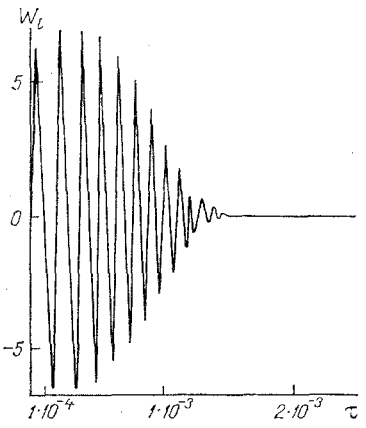

Fig. 3

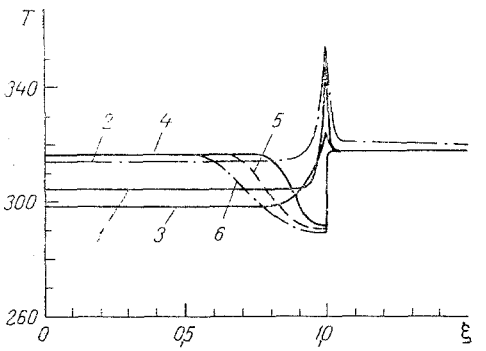

Fig. 4

Fig. 3. Change in the mass velocity of the liquid over time at the bubble boundary with T_0 = 300 K and a pressure drop from 10^6 to $5\cdot10^5$ Pa.

Fig. 4. Profile of the temperature of a pulsating vapor bubble with an initial temperature T_0 = 318 K and a pressure drop from $1.5\cdot10^6$ Pa to $1\cdot10^6$ Pa at different moments of time: 1) $\tau = 0.5\cdot10^{-3}$ sec; 2) $0.1\cdot10^{-2}$ sec; with a pressure drop from 10^6 to $5\cdot10^5$ Pa: 3) $\tau = 0.5\cdot10^{-3}$ sec; with a pressure drop from 10^5 to $8\cdot10^4$ Pa: 4) $\tau = 0.5\cdot10^{-3}$ sec; 5) $0.1\cdot10^{-2}$; 6) $0.2\cdot10^{-2}$ sec. T, K.

Heat transfer is most intense in the immediate vicinity of the boundary of the vapor bubble as it oscillates. Figure 2 shows the profile of velocity V_V inside the vapor bubble at T_0 = 300 K with a pressure drop from $P_V = 10^6$ to $P_\ell = 5\cdot10^5$ Pa. It is evident that the intensity of the processes occurring at the center of the bubble is considerably lower than at its boundary (the boundary corresponds to $\xi = 1$).

Due to the intensive occurrence of phase transformations between the liquid and vapor phases at and near the boundary of the bubble, the change in the mass velocity of the phases at the boundary is oscillatory in character. The amplitude of the oscillations gradually decreases until equilibrium is established. Figure 3 shows the dependence of the mass velocity of the liquid at the bubble boundary on time at T_0 = 300 K and with a pressure drop from 10^6 to $5\cdot10^5$ Pa.

The change in the temperature of the vapor inside the bubble during its pulsations depends heavily on the initial temperature T_0, the initial vapor pressure P_V, and the pressure drop. Figure 4 shows temperature profiles of a pulsating vapor bubble for T_0 = 318 K and different values of pressure drop at certain moments of time. Comparison of curves 1, 3, and 4, calculated for $t = 0.5\cdot10^{-3}$ sec, shows that they all have jumps near the boundary of the bubble. However, the forms of these curves differ for high and low initial pressures P_V. The same conclusion can be reached by comparing curves 2 and 5 for $t = 0.1\cdot10^{-2}$ sec.

The completed calculations show that the dynamics of cavitational processes are heavily dependent on heat transfer in the two-phase vapor–liquid mixture in the presence of liquid–vapor phase transformations. The results of the calculations make it possible to predict the behavior of cavitation bubbles in a pipe filled with a heat carrier undergoing depressurization in a vacuum.

NOTATION

C_{pV}, C_ℓ, specific heat capacities (at constant pressure) of the vapor and liquid; ρ_V, ρ_ℓ, densities; T_V, T_ℓ, temperatures; V_V, V_ℓ, velocities; P_V, P_ℓ, pressures; W_V, W_ℓ, mass velocities on the bubble surface; λ_V, λ_ℓ, thermal conductivities of the vapor and liquid; γ, adiabatic exponent of the vapor; σ, surface tension; ν, kinematic viscosity; j, rate of phase transformations; m, mass; ℓ, heat of phase transformation. Indices: v, vapor phase; ℓ, liquid phase.

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TOWARD A THEORY OF TRANSPORT IN HETEROGENEOUS MEDIA

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A unified equation is obtained for description of nonsteady state heat and mass transport in two-phase heterogeneous media in the low- and high-frequency approximations. Heating of a granular bed by a solid wall is considered as an example.

1. The basis of the traditional method of describing heat and mass transport processes in disperse and other heterogeneous materials is a system of equations for moderate temperatures or impurity concentrations in the individual phases, which consider interphase heat and mass exchange. Such equations are usually derived by the use of semiempirical relationships [1-3], although their general structure has been confirmed by results of a stricter analysis [4, 5]. We will write this system for heat transport in situations where convective and dispersive transport is significant for only one (continuous) phase:

$$\varepsilon d_1 c_1 \left(\frac{\partial}{\partial t} + \mathbf{u}\nabla\right) T_1 = \lambda_* \Delta T_1 - \beta (T_1 - T_2),$$
$$(1 - \varepsilon) d_2 c_2 \frac{\partial T_2}{\partial t} = \beta (T_1 - T_2). \qquad (1)$$

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